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1988 J. Phys. A: Math. Gen. 21 4235

(http://iopscience.iop.org/0305-4470/21/22/024)

J. Phys. A: Math. Gen. **21** (1988) 4235-4240. Printed in the UK

Aggregation of particles which move on deterministic trajectories with fractal dimension two: I. A simple and new model for DLA

Yoshi-hiro Taguchi

Department of Physics, Tokyo Institute of Technology, Oh-okayama, Meguro-ku, Tokyo 152, Japan

Received 15 February 1988

Abstract. We simulate a new type of aggregation process: particles follow deterministic trajectories. The trajectories have a fractal dimension, d_w, of two. d_w is defined as $S(r) \sim r^{d_w}$, where $S(r)$ is the total number of sites a particle visits while travelling a distance r. When the growth process is started with a single stationary particle on a square lattice, a large aggregate looks like diffusion-limited aggregation (DLA). The fractal dimension, d_f, of the aggregate is nearly equal to 1.7; that of DLA is 1.67 ± 0.05. Moreover, the aggregate has axial anisotropy, although it includes less than one thousand particles. Therefore the aggregation belongs to the same universality class of DLA and is supposed to be the highly noise-reduced model for DLA. Because the Laplace equation has no relationship to the aggregation process, the Laplace equation does not seem to be a necessary condition for producing aggregates like DLA.

Diffusion-limited aggregation (DLA) is of major interest because it is a simple model for several physical phenomena: metal leaves, viscous fingers, dielectric breakdowns, etc. In all of the phenomena, growth probabilities depend on the gradient of the solution of the Laplace equations. On the other hand, DLA and these physical phenomena show fractal patterns. Hence much effort has been made to understand the relationship between the Laplace equation and the fractal pattern. However, in dimensional analysis—one of successive treatments (Tokuyama and Kawasaki 1984, Honda *et al* 1986, Matsushita *et al* 1986)—the Laplace equation is not taken into account at all. These results are

$$d_f = \frac{d^2+1}{d+1} \tag{1}$$

(Tokuyama and Kawasaki 1984) and

$$d_f = \frac{d^2+d_w-1}{d+d_w-1} \tag{2}$$

(Honda *et al* 1986, Matsushita *et al* 1986) where d_f is the fractal dimension of DLA and d is the dimension of the lattice; d_w is the fractal dimension of the trajectories of particles, with $d_w = 2$ for DLA. Actually, they use only $d_w = 2$ in dimensional analysis for DLA. However, equations (1) and (2) agree with numerical simulations (Meakin 1983, 1984, 1985). Does this fact suggest that d_f depends upon only the fractal dimension d_w of the trajectories of particles? Does not the Laplace equation directly relate to the fractal dimension d_f? If equations (1) and (2) are exact, all of the trajectories of $d_w = 2$ reduce to the aggregates which obey equations (1) and (2). To make this point clear, we consider trajectories which are not diffusions but have d_w equal to 2 and the aggregation d_f is measured when the particles have such trajectories.

Before presenting the calculations we must outline the meaning of d_w. We define the total number of visited sites as $S(r)$, where r is the travelling distance of the particle. Then d_w is defined as follows:

$$S(r) \sim r^{d_w}. \tag{3}$$

d_w differs from d_w^*, which is defined as follows:

$$[r(t)]^2 \sim t^{2/d_w^*} \tag{4}$$

where t is the total number of steps. In general, $d_w^* \geq d_w$†. During the aggregation process of DLA, a particle sticks to the cluster when it comes into contact with a stationary particle in the cluster for the first time. Hence, it is not worthwhile counting the number of times that a particle visits a site. Therefore we should consider d_w instead of d_w^*.

In the present paper we deal with fractal trajectories which are deterministic, because they are easier to draw than statistical ones: random walks, Lévy flights, etc. We shall call aggregations limited by such deterministic trajectories 'deterministic diffusion-limited aggregation' (DDLA). The first example of DDLA is Peano-curve-limited aggregation (PCLA). The Peano curve is one of the fractal figures which completely cover two-dimensional space. Figure 1 shows the initiator and the generator of the Peano curve used in our study. Although d_w^* for this curve is $\ln 6/\ln 2$, d_w is 2. The simulation procedure is almost the same as the ordinary procedure for DLA. We start with a square lattice and occupy a site with a seed particle. A site is then selected on the perimeter of a 'launching circle' which has a radius of $r_{max}+2$ lattice units, where r_{max} is the maximum radius for the cluster (initially, $r_{max}=0$). A 'direction' for the initiator is randomly chosen among four directions and a particle is released along the Peano curve. If a mobile particle contacts the cluster it sticks at that point. Additional particles are added in the same way. Figure 2 shows a cluster of 795 particles which looks like DLA. The fractal dimension, d_f, calculated with the radius of gyration is 1.68; that of DLA is 1.67 ± 0.05 (Meakin 1983). It seems that the d_f of PCLA is the same as that of DLA. Moreover, figure 2 shows a slight diamond shape,

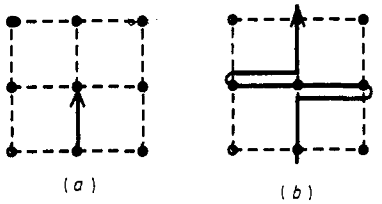

Figure 1. The initiator and generator of the Peano curve on the square lattice: (a) the initiator and (b) the generator. Small circles are sites. The arrow represents the direction of trajectories. d_w^* (see equation (4)) is $\ln 6/\ln 2$, but d_w (see equation (3)) is two.

† d_w is equivalent to the spreading dimension (Rammal *et al* 1984, Berker and Ostlund 1979) or the connectivity dimension (Suzuki 1983). However we should take note of the fact that d_w in the present paper is defined on trajectories. The spreading and connectivity dimensions are on lattices.

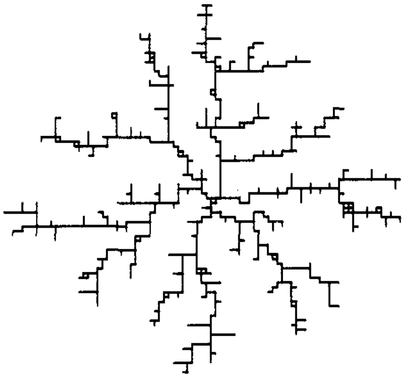

Figure 2. A cluster of 795 particles generated using Peano-curve-limited aggregation (PCLA). $d_f = 1.68$. It looks like DLA and has a slightly diamond shape.

i.e. axial anisotropy. To confirm this point, we adopt a new launching circle. On the square lattice, the distance between two sites should be measured with the minimum number of steps to travel from one site to another. Therefore the radius of the launching circle should also be measured by it. Then the launching circle is replaced with a 'launching diamond' (see figure 3). Using the launching diamond, we obtain the pattern shown in figure 4. d_f is also calculated with the number of steps instead of the radius of gyration, i.e.

$$\bar{r}(N) \equiv \frac{1}{N} \sum_{(x,y)\in\text{cluster}} |x| + |y| \tag{5}$$

is plotted against N, where N is the cluster size and the summation runs over all of

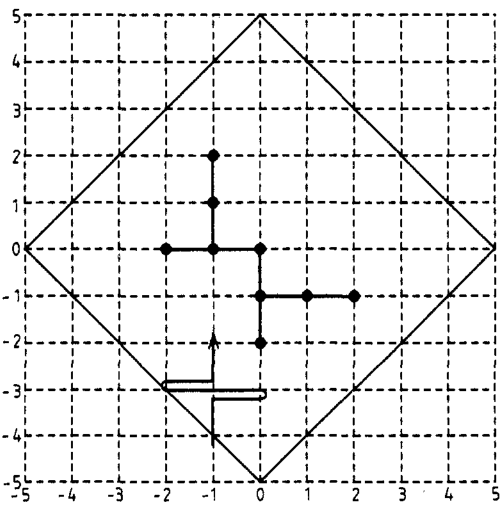

Figure 3. The explanation of 'launching diamond' and PCLA. The large square is a launching diamond of $\bar{r}_{max} + 2 = 5$. A seed particle is located at a site (0, 0). The arrow represents the direction of the trajectory. The first six steps of the Peano curve are also illustrated. The next aggregated site is (0, −3).

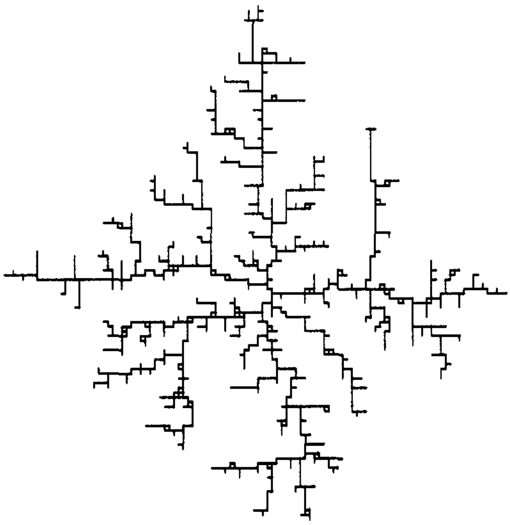

Figure 4. A cluster of 1185 particles of PCLA using the launching diamond. $d_f = 1.72$ (see figure 5). It apparently has a diamond shape, i.e. the axial anisotropy.

the particles in the cluster. Figure 5 gives us a d_f of 1.72, which is the same as that of DLA. Remarkably, a log N-log $\bar{r}$ plot fits a straight line very well, although the total number of particles is about 10^3. In addition to this, we try two more kinds of deterministic trajectories (see figure 6). We shall call figure 6(a) a square vortex and figure 6(b) a diamond vortex. The d_w of both of them is also two. Figure 7(a) shows square-vortex-limited aggregation (SVLA) and figure 7(b) shows diamond-vortex-limited aggregation (DVLA). They also have diamond shapes and d_f is about 1.7 (see

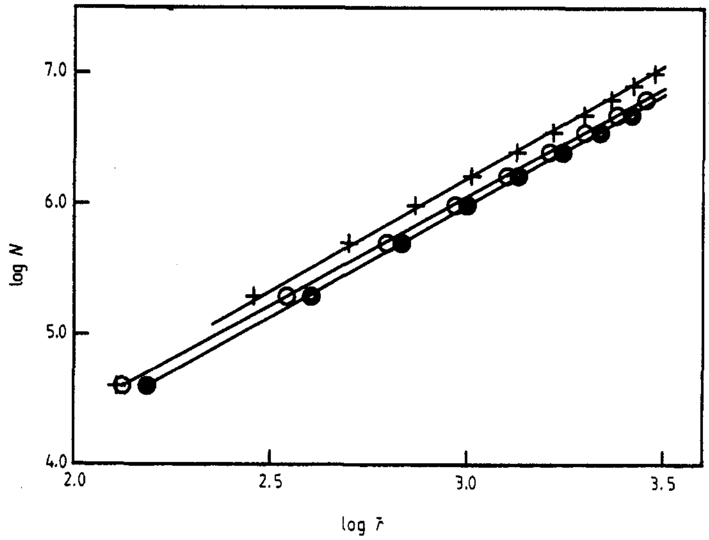

Figure 5. Log N-log $\bar{r}$ plots for deterministic diffusion-limited aggregations (DDLA). Crosses for PCLA ($d_f = 1.72$), concentric circles for SVLA (see figure 7(a): $d_f = 1.70$) and open circles for DVLA (see figure 7(b): $d_f = 1.66$). Straight lines adopt d_f respectively.

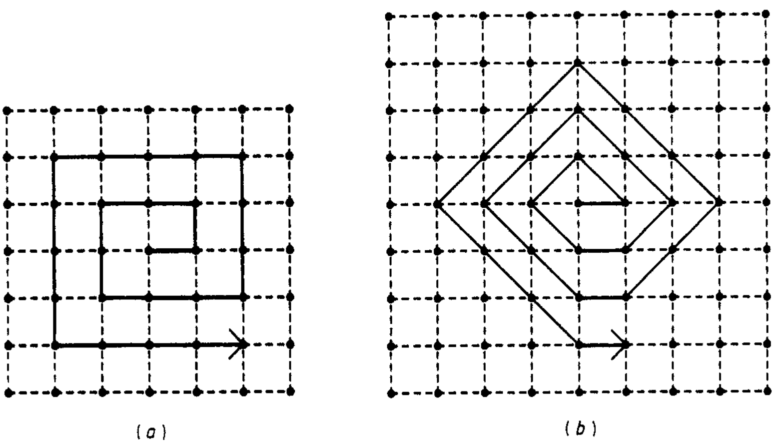

Figure 6. The vortex-type deterministic trajectories on the square lattice: (*a*) the first 24 steps of the square vortex and (*b*) the first 25 steps of the diamond vortex. The arrows represent directions of trajectories.

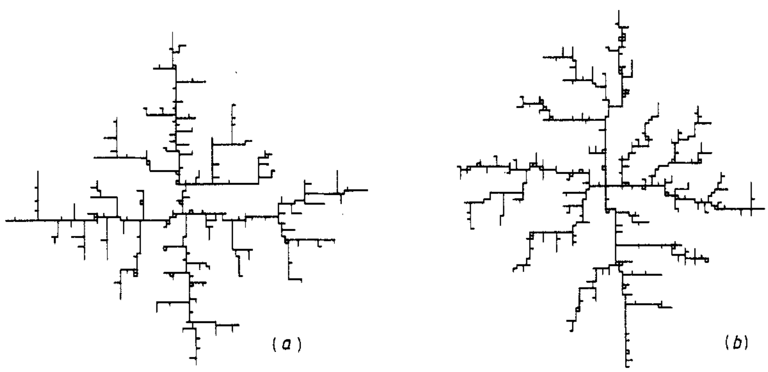

Figure 7. (*a*) A cluster of 864 particles generated using square-vortex-limited aggregation (SVLA), $d_f = 1.70$. (*b*) A cluster of 958 particles generated using diamond-vortex-limited aggregation (DVLA), $d_f = 1.66$. Both of them apprently have the diamond shape, i.e. axial anisotropy.

figure 5). These results suggest that DDLA belongs to the same universality class of DLA. Because DDLA has no relationship to the Laplace equation, the Laplace equation does not seem to be a necessary condition for producing DLA-like clusters. Moreover, DDLA is probably a highly noise-reduced model of DLA for two reasons.

(i) The log N-log $\tilde{r}$ plots fit straight lines very well for N in the range 10^2–10^3. DLA cannot supply such a good fit for the same cluster size.

(ii) DDLA has the diamond shape even if the cluster size is less than 10^3. Such an effect cannot be observed until the cluster grows to the size of 10^6 (Meakin *et al* 1987).

Hence DDLA is a useful tool for investigating the behaviour of DLA for large clusters.

To conclude, we propose a new type of aggregation process, DDLA. DDLA belongs to the same universality class of DLA. Because DDLA has no relationship to the Laplace equation, the Laplace equation does not seem to be a necessary condition for producing DLA-like clusters.

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